

Exclusive and inclusive muon pair production in collisions of relativistic nuclei

K. Hencken

Institut für Physik, Universität Basel, 4056, Basel, Switzerland

E.A. Kuraev

Joint Institute of Nuclear Research, 141980, Dubna, Russia

V.G. Serbo

Novosibirsk State University, 630090, Novosibirsk, Russia

(Dated: May 31, 2006)

Abstract

The exclusive production of one $\mu^+\mu^-$ pair in collisions of two ultra-relativistic nuclei is considered. We present a simple method for the calculation of the Born cross section for this process based on an improved equivalent photon approximation. We find that the Coulomb corrections to this cross section (corresponding to multi-photon exchange of the produced μ^\pm with the nuclei) are small while the unitarity corrections (corresponding to the exchange of light-by-light blocks between nuclei) are large. This is in sharp contrast to the exclusive e^+e^- pair production where the Coulomb corrections to the Born cross section are large while the unitarity corrections are small. We calculate also the cross section for the production of one $\mu^+\mu^-$ pair and several e^+e^- pairs in the leading logarithmic approximation. Using this cross section we find that the inclusive production of $\mu^+\mu^-$ pair coincides in this approximation with its Born value.

PACS numbers:

I. INTRODUCTION

Lepton pair production in ultra-relativistic nuclear collisions were discussed in numerous papers (see [1] for a review and references therein). For the RHIC and LHC colliders the charge numbers of nuclei $Z_1 = Z_2 \equiv Z$ and their Lorentz factors $\gamma_1 = \gamma_2 \equiv \gamma$ are given in Table I.

TABLE I: Colliders and cross sections for the lepton pair production

Collider	Z	γ	$\sigma_{\text{Born}}^{e^+e^-}$ [kb]	$\sigma_{\text{Born}}^{\mu^+\mu^-}$ [b]
RHIC, Au-Au	79	108	36.0	0.23
LHC, Pb-Pb	82	3000	227	2.6
LHC, Ar-Ar	18	3400	0.554	0.0082

The cross section of one e^+e^- pair production in Born approximation, described by the Feynman diagram of Fig. 1, was obtained many years ago [2]. Since the Born cross section $\sigma_{\text{Born}}^{e^+e^-}$ is huge (see Table I), the e^+e^- pair production can be a serious background for many experiments. It is also an important issue for the beam lifetime and luminosity of these colliders [3]. It means that various corrections to the Born cross section, as well as, the cross section for n -pair production, are of great importance. At present, there are a lot of controversial and incorrect statements in papers devoted to this subject. The corresponding references and critical remarks can be found in Refs. [1, 4, 5].

Since the parameter $Z\alpha$ may be not small ($Z\alpha \approx 0.6$ for Au-Au and Pb-Pb collisions), the whole series in $Z\alpha$ has to be summed in order to obtain the cross section with sufficient accuracy. The exact cross section for one pair production σ_1 can be split into the form

$$\sigma_1 = \sigma_{\text{Born}} + \sigma_{\text{Coul}} + \sigma_{\text{unit}}, \quad (1)$$

where two different types of corrections need to be distinguished. The Coulomb correction σ_{Coul} corresponds to multi-photon exchange of the produced e^\pm with the nuclei (Fig. 2); it was calculated in [4]. The unitarity correction σ_{unit} corresponds to the exchange of light-by-light blocks between nuclei (Fig. 3); it was calculated in [5]. It was found in [4]

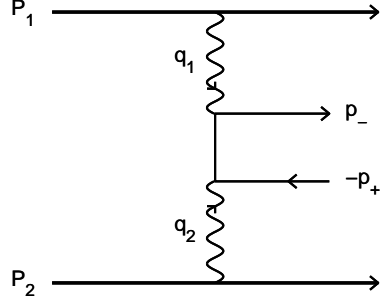


FIG. 1: The Feynman diagram for the lepton pair production in the Born approximation

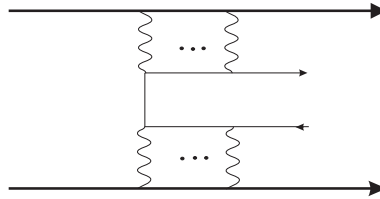


FIG. 2: The Feynman diagram for the Coulomb correction

and [5] that the Coulomb corrections are large while the unitarity corrections are small (see Table II). The results of [5] were confirmed recently in [6] by a direct summation of the Feynman diagrams.

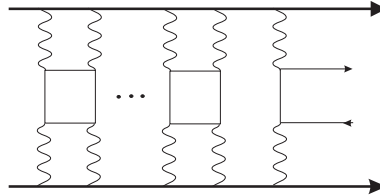


FIG. 3: The Feynman diagram for the unitarity correction

In this paper we present our calculations related to the exclusive and inclusive muon pair production. This process may be easier to observe experimentally than e^+e^- pair production described above. It should be stressed that the calculation scheme, as well as, the final results for the $\mu^+\mu^-$ pair production are quite different than those for the e^+e^- pair production.

In the next section we calculate the Born cross section for one $\mu^+\mu^-$ pair production using the improved equivalent photon approximation with an accuracy about 5 %. In Sect. 3 we present the Coulomb and unitarity corrections to the exclusive production

TABLE II: Coulomb and unitarity corrections to the e^+e^- pair production

Collider	$\frac{\sigma_{\text{Coul}}}{\sigma_{\text{Born}}}$	$\frac{\sigma_{\text{unit}}}{\sigma_{\text{Born}}}$
RHIC, Au-Au	-25%	-4.1%
LHC, Pb-Pb	-14%	-3.3%
LHC, Ar-Ar	-1.06%	-0.025%

of one muon pair. In this section we also obtain the probability for the production of one muon and several electron pairs in collisions of nuclei at a given impact parameter ρ . This result allows us to calculate the inclusive production of one muon pair in Sect. 4. In Sect. 5 we calculate the cross section for two muon pair production in lowest order. The conclusion of the paper is given in Sect. 6. In the Appendix we consider the relatively simple process of one muon pair production by a real photon off a nucleus $\gamma Z \rightarrow Z\mu^+\mu^-$ which serves as a good test of the approach used. A short preliminary version of this paper was published in [7].

II. BORN CROSS SECTION FOR ONE $\mu^+\mu^-$ PAIR PRODUCTION

The production of one $\mu^+\mu^-$ pair

$$Z_1 + Z_2 \rightarrow Z_1 + Z_2 + \mu^+\mu^- \quad (2)$$

in the Born approximation is described by the Feynman diagram of Fig. 1. When two nuclei with charges Z_1e and Z_2e and 4-momenta P_1 and P_2 collide with each other, they emit equivalent (virtual) photons with the 4-momenta q_1, q_2 , energies ω_1, ω_2 and their virtualities $Q_1^2 = -q_1^2, Q_2^2 = -q_2^2$. Upon fusion, these photons produce a $\mu^+\mu^-$ pair with the total four-momentum $q_1 + q_2$ and the invariant mass squared $W^2 = (q_1 + q_2)^2$. Besides this we denote

$$(P_1 + P_2)^2 = 4E^2 = 4M^2\gamma^2, \quad \alpha \approx 1/137$$

and use the system of units in which $c = 1$ and $\hbar = 1$.

A. General formulae

The Born cross section of the process (2) can be calculated to a good accuracy using the equivalent photon approximation (EPA) in the improved variant presented, for example, in Ref. [8]. Let the numbers of equivalent photons be dn_1 and dn_2 . The most important contribution to the production cross section stems from photons with very small virtualities $Q_i^2 \ll \mu^2$ where μ is the muon mass. Therefore to a good approximation, the photons move in opposite directions, and $W^2 \approx 4\omega_1\omega_2$. In this very region the Born differential cross section $d\sigma_B$ for the process considered is related to the cross section $\sigma_{\gamma\gamma}$ for the process with real photons: $\gamma\gamma \rightarrow \mu^+\mu^-$ by the equation

$$d\sigma_B = dn_1 dn_2 d\sigma_{\gamma\gamma}(W^2), \quad W^2 \approx 4\omega_1\omega_2. \quad (3)$$

The number of equivalent photons are (see Eq. (D.4) in Ref. [8])

$$dn_i(\omega_i, Q_i^2) = \frac{Z_i^2 \alpha}{\pi} \left(1 - \frac{\omega_i}{E_i}\right) \frac{d\omega_i}{\omega_i} \left(1 - \frac{Q_{i\min}^2}{Q_i^2}\right) F^2(Q_i^2) \frac{dQ_i^2}{Q_i^2}, \quad (4)$$

where

$$Q_i^2 \geq Q_{i\min}^2 = \frac{\omega_i^2}{\gamma^2} \quad (5)$$

and $F(Q^2)$ is the electromagnetic form factor of the nucleus. It is important that the integral over Q^2 converges rapidly for $Q^2 > 1/R^2$, where

$$R = 1.2 A^{1/3} \text{ fm} \quad (6)$$

is the radius of the nucleus with $A \approx M/m_p$ the number of nucleons ($R \approx 7$ fm, $1/R \approx 28$ MeV for Au and Pb). Since $Q_{\min}^2 \lesssim 1/R^2$, the main contribution to the cross section is given by virtual photons with energies

$$\omega_i \lesssim \gamma/R. \quad (7)$$

Therefore, we can use the spectrum of equivalent photons neglecting terms proportional to ω_i/E_i given by:

$$dn_i(\omega_i, Q_i^2) = \frac{Z_i^2 \alpha}{\pi} \frac{d\omega_i}{\omega_i} \left(1 - \frac{\omega_i^2}{\gamma^2 Q_i^2}\right) F^2(Q_i^2) \frac{dQ_i^2}{Q_i^2}. \quad (8)$$

After the transformation

$$\frac{d\omega_1}{\omega_1} \frac{d\omega_2}{\omega_2} = \frac{d\omega_1}{\omega_1} \frac{dW^2}{W^2} \quad (9)$$

we cast the cross section in the form

$$d\sigma_{\text{B}} = \frac{Z_1^2 Z_2^2 \alpha^2}{\pi^2} \frac{d\omega_1}{\omega_1} \left(1 - \frac{\omega_1^2}{\gamma^2 Q_1^2}\right) F^2(Q_1^2) \frac{dQ_1^2}{Q_1^2} \times \left(1 - \frac{\omega_2^2}{\gamma^2 Q_2^2}\right) F^2(Q_2^2) \frac{dQ_2^2}{Q_2^2} \frac{dW^2}{W^2} \sigma_{\gamma\gamma}(W^2), \quad (10)$$

where $\omega_2 \approx W^2/(4\omega_1)$.

B. Leading logarithmic approximation (LLA)

Before using the calculation scheme above, it is instructive to present a rougher but simpler approximation — the so called leading logarithmic approximation (LLA). In the LLA, the equivalent photon spectrum as a function of photon energy $dn_i(\omega_i)$ is obtained after integrating $dn_i(\omega_i, Q_i^2)$ over Q_i^2 in the region between

$$Q_{i\text{ min}}^2 \leq Q_i^2 \lesssim 1/R^2 \quad (11)$$

which leads to

$$dn_i(\omega_i) \approx \frac{Z_i^2 \alpha}{\pi} \ln \frac{\gamma^2}{(R\omega_i)^2} \frac{d\omega_i}{\omega_i}. \quad (12)$$

The restriction $Q_{i\text{ min}}^2 \lesssim 1/R^2$ corresponds to the integration interval

$$a = \frac{W^2 R}{4\gamma} \lesssim \omega_1 \lesssim b = \frac{\gamma}{R}, \quad (13)$$

which gives

$$\sigma_{\text{B}}^{\text{LLA}} = \frac{Z_1^2 Z_2^2 \alpha^2}{\pi^2} \int_{4\mu^2}^{\infty} \frac{dW^2}{W^2} \sigma_{\gamma\gamma}(W^2) \int_a^b \frac{d\omega_1}{\omega_1} \ln \frac{b^2}{\omega_1^2} \ln \frac{\omega_1^2}{a^2}. \quad (14)$$

Since $\sigma_{\gamma\gamma}(W^2) \approx (4\pi\alpha^2/W^2) \ln(W^2/\mu^2)$ for large values of $W \gg \mu$, the main contribution to the Born cross section comes from the region of small values of W near the threshold. Therefore, within logarithmic accuracy we replace W by some fixed value $W_0 \sim 2\mu$ in the lower bound a . After that the integral over W^2 gives

$$I = \int_{4\mu^2}^{\infty} \frac{dW^2}{W^2} \sigma_{\gamma\gamma}(W^2) = \frac{14\pi\alpha^2}{9\mu^2} \quad (15)$$

and further integration over ω_1 leads to

$$\int_a^b \frac{d\omega_1}{\omega_1} \ln \frac{b^2}{\omega_1^2} \ln \frac{\omega_1^2}{a^2} = \frac{2}{3} L^3, \quad (16)$$

where

$$L = \ln \frac{\gamma^2}{(W_0 R/2)^2}. \quad (17)$$

As a result, we obtain

$$\sigma_B^{\text{LLA}} = \frac{28}{27\pi} \frac{(Z_1 \alpha Z_2 \alpha)^2}{\mu^2} L^3 \quad (18)$$

in accordance with the result of Landau & Lifshitz [2]. The accuracy of the LLA depends on the choice of the value for W_0 . If we use for numerical estimations $W_0 = 3\mu$, then the accuracy of the LLA for the colliders discussed is about 15%.

The same result can be obtained in the framework of the impact parameter dependent representation, which will also be useful later. For this aim we introduce the probability for muon pair production $P_B(\rho)$ in the collision of two nuclei at a fixed impact parameter ρ . For $\gamma \gg 1$, it is possible to treat the nuclei as sources of external fields and to calculate $P_B(\rho)$ analytically using the same approach as in Ref. [5]. The Born cross section σ_B can then be obtained by the integration of $P_B(\rho)$ over the impact parameter:

$$\sigma_B = \int P_B(\rho) d^2\rho. \quad (19)$$

We calculate this probability in the LLA using Eq. (3) with

$$dn_i = \frac{Z_i^2 \alpha}{\pi^2} \frac{d\omega_i}{\omega_i} \frac{d^2\rho_i}{\rho_i^2}; \quad \omega_i \ll \frac{\gamma}{R}; \quad R \ll \rho_i \ll \frac{\gamma}{\omega_i}, \quad (20)$$

where ρ_i is the impact parameter of i -th equivalent photon with respect to the i -th nucleus.

This allows us to write the above probability in the form

$$P_B(\rho) = \int dn_1 dn_2 \delta(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2 - \boldsymbol{\rho}) \sigma_{\gamma\gamma}(W^2) = \frac{28}{9\pi^2} \frac{(Z_1 \alpha Z_2 \alpha)^2}{(\mu\rho)^2} \Phi(\rho). \quad (21)$$

Depending on the value of ρ two different forms for $\Phi(\rho)$ need to be used:

$$\Phi(\rho) = \left(4 \ln \frac{\gamma}{\mu\rho} + \ln \frac{\rho}{R}\right) \ln \frac{\rho}{R} \quad \text{for} \quad R \ll \rho \leq \frac{\gamma}{\mu}, \quad (22)$$

$$\Phi(\rho) = \left(\ln \frac{\gamma^2}{\mu^2 \rho R}\right)^2 \quad \text{for} \quad \frac{\gamma}{\mu} \leq \rho \ll \frac{\gamma^2}{\mu^2 R}. \quad (23)$$

Note that the function $\Phi(\rho)$ is continuous at $\rho = \gamma/\mu$ together with its first derivative. As expected the integration of $P_B(\rho)$ over $\boldsymbol{\rho}$ in the region $R < \rho < \gamma^2/(\mu^2 R)$ gives back the result in (18).

To prove (21)–(23), we make the transformation given in (9) together with the integration over W^2 according to (15). This gives

$$P_B(\rho) = \frac{14}{9\pi^3} \frac{(Z_1 \alpha Z_2 \alpha)^2}{\mu^2} \int_{\mu^2 R/\gamma}^{\gamma/R} \frac{d\omega_1}{\omega_1} \int \frac{d^2\rho_1}{\rho_1^2 (\boldsymbol{\rho} - \boldsymbol{\rho}_1)^2} \vartheta\left(\frac{\gamma}{\omega_1} - \rho_1\right) \vartheta\left(\frac{\gamma\omega_1}{\mu^2} - |\boldsymbol{\rho} - \boldsymbol{\rho}_1|\right) \quad (24)$$

where $\vartheta(x)$ is the step function. The main contribution to this integral is given by two regions: $R \ll \rho_1 \ll \rho$ and $R \ll |\boldsymbol{\rho} - \boldsymbol{\rho}_1| = \rho_2 \ll \rho$. Moreover, the two regions in the ω_1 integration with $\mu < \omega_1 < \gamma/R$ and $\mu^2 R/\gamma < \omega_1 < \mu$ give the same contributions. As a result we get

$$\begin{aligned}\Phi(\rho) &= 2(J_1 + J_2), \\ J_1 &= \int_{\mu}^{\gamma/R} \frac{d\omega_1}{\omega_1} \int_R^{\rho} \frac{d\rho_1}{\rho_1} \vartheta(\gamma/\omega_1 - \rho_1) \vartheta(\gamma\omega_1/\mu^2 - \rho), \\ J_2 &= \int_{\mu}^{\gamma/R} \frac{d\omega_1}{\omega_1} \int_R^{\rho} \frac{d\rho_2}{\rho_2} \vartheta(\gamma/\omega_1 - \rho) \vartheta(\gamma\omega_1/\mu^2 - \rho_2).\end{aligned}\tag{25}$$

Next we consider the two regions of ρ .

In the region of relatively small impact parameters, $R \ll \rho \leq \gamma/\mu$, the second step function does not impose any limitations, therefore,

$$\begin{aligned}J_1 &= \int_{\mu}^{\gamma/\rho} \frac{d\omega_1}{\omega_1} \int_R^{\rho} \frac{d\rho_1}{\rho_1} + \int_{\gamma/\rho}^{\gamma/R} \frac{d\omega_1}{\omega_1} \int_R^{\gamma/\omega_1} \frac{d\rho_1}{\rho_1} = \ln \frac{\gamma}{\mu\rho} \ln \frac{\rho}{R} + \frac{1}{2} \left(\ln \frac{\rho}{R} \right)^2 \\ J_2 &= \int_{\mu}^{\gamma/\rho} \frac{d\omega_1}{\omega_1} \int_R^{\rho} \frac{d\rho_1}{\rho_1} = \ln \frac{\gamma}{\mu\rho} \ln \frac{\rho}{R}.\end{aligned}$$

Summing up, we obtain (22)

In the region of relatively large impact parameters, $\gamma/\mu \leq \rho \ll \gamma^2/(\mu^2 R)$, we have

$$J_1 = \int_{\mu^2 \rho/\gamma}^{\gamma/R} \frac{d\omega_1}{\omega_1} \int_R^{\gamma/\omega_1} \frac{d\rho_1}{\rho_1} = \frac{1}{2} \left(\ln \frac{\gamma^2}{\mu^2 R \rho} \right)^2, \quad J_2 = 0,$$

therefore, the sum gives (23).

We compare Eqs. (21)–(23) for $\Phi(\rho)$ with the numerical calculations based on the exact matrix element calculated with the approach as outlined in [9]. We find good agreement for Pb-Pb collisions: the discrepancy is less than 10 % at $\mu\rho > 10$ and it is less than 15 % at $\mu\rho > 2\mu R = 7.55$.

C. More refined calculation

In the calculation below we use for the form factor of the nucleus the simple approximation of a monopole form factor, which corresponds to an exponentially decreasing charge distribution, whose mean squared radius $\sqrt{\langle r^2 \rangle}$ is adjusted to the experimental value:

$$F(Q^2) = \frac{1}{1 + Q^2/\Lambda^2}, \quad \Lambda^2 = \frac{6}{\langle r^2 \rangle}.\tag{26}$$

For lead and gold, the parameter is $\Lambda \approx 80 \text{ MeV}$. This approximate form of the form factor enables us to perform some calculations analytically, which otherwise could only be done numerically.

The equivalent photon spectrum $dn_i(\omega_i)$ is obtained after integrating $dn_i(\omega_i, Q_i^2)$ over Q_i^2 (the upper limit of this integration can be set to be equal to infinity in a good approximation, due to the fast convergence of the integral at $Q^2 > \Lambda^2$):

$$dn_i(\omega_i) = \frac{Z_i^2 \alpha}{\pi} f\left(\frac{\omega_i}{\Lambda \gamma}\right) \frac{d\omega_i}{\omega_i}. \quad (27)$$

Here the function

$$f(x) = (1 + 2x^2) \ln\left(\frac{1}{x^2} + 1\right) - 2 \quad (28)$$

is large for small values of x ,

$$f(x) \approx \ln \frac{1}{x^2} - 2 = \ln \frac{1}{(ex)^2} \text{ at } x \ll 1, \quad (29)$$

but drops very quickly for large x in accordance with Eq. (7):

$$f(x) < \frac{1}{6x^4} \text{ for } x > 1. \quad (30)$$

Finally we obtain

$$\sigma_B = \frac{Z_1^2 Z_2^2 \alpha^2}{\pi^2} \int_{4\mu^2}^{\infty} \frac{dW^2}{W^2} G(W^2) \sigma_{\gamma\gamma}(W^2) = \frac{(Z_1 \alpha Z_2 \alpha)^2}{\pi \mu^2} J(\gamma \Lambda / \mu), \quad (31)$$

where

$$G(W^2) = \int_{\omega_{\min}}^{\omega_{\max}} \frac{d\omega}{\omega} f\left(\frac{\omega}{\Lambda \gamma}\right) f\left(\frac{W^2}{4\Lambda \gamma \omega}\right). \quad (32)$$

Since $\omega_i < E$ and $\omega_1 \omega_2 \sim \mu^2$ we have $\omega_{\min} \sim \mu^2/E$ and $\omega_{\max} = E$. However, due to the fast decrease of $f(x)$ for $x > 1$ one can extend these limits up to $\omega_{\min} = 0$ and $\omega_{\max} = \infty$ without any lack of accuracy, therefore,

$$G(W^2) = 2 \int_0^{\infty} f(x_1) f(x_2) dy, \quad x_{1,2} = \frac{W}{2\Lambda \gamma} e^{\pm y}. \quad (33)$$

A numerical evaluation of the integrals in Eqs. (31), (32) yields the function $J(\gamma \Lambda / \mu)$ presented in Fig. 4.

Corrections to the photon spectrum are represented by terms in Eqs. (3), (8) of the order of Q_i^2/W^2 (see Eqs. (E.1) in Ref. [8]), which are dropped before the integration over

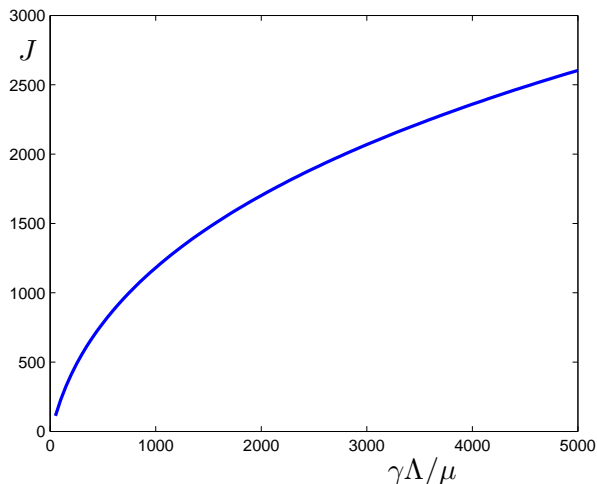


FIG. 4: The function $J(\gamma\Lambda/\mu)$ from Eq. (31)

Q_i^2 is done. After the integration with weight $1/Q_i^2$ the relative value of these corrections becomes of the order of

$$\eta_1 = \frac{\Lambda^2}{W^2 L}. \quad (34)$$

Thus, for the collisions considered here one can estimate the accuracy of the calculations on the level $\eta_1 \sim 5\%$. Another test of accuracy of the approach used is given in Appendix.

Note that in the LLA the function $G(W^2)$ is just

$$G^{\text{LLA}}(W^2) = \frac{2}{3} \left[\ln \frac{\gamma^2}{(eW/2\Lambda)^2} \right]^3 \approx \frac{2}{3} \left[\ln \frac{\gamma^2}{(WR/2)^2} \right]^3 \quad (35)$$

in accordance with Eq. (16) (taking into account that $\Lambda/e \approx 1/R$).

III. COULOMB AND UNITARITY CORRECTIONS

The Coulomb correction corresponds to Feynman diagram of Fig. 2. Due to the restriction of the transverse momenta of additionally exchanged photons to the range below $1/R$, the effective parameter of the perturbation series is not $(Z\alpha)^2$ but $(Z\alpha)^2/(R\mu)^2$. In addition, the contribution of the additional photons is suppressed by a logarithmic factor. Indeed, the cross section for two-photon production mechanism is proportional to L^3 , while the cross section for the multiple-photon production mechanism is proportional only to L^2 . Therefore, the real parameter describing the suppression of the Coulomb

correction is of the order of

$$\eta_2 = \frac{(Z\alpha)^2}{(R\mu)^2 L} \quad (36)$$

which corresponds to Coulomb corrections of less than 1 %. The example considered in Appendix confirms this estimate.

The unitarity correction σ_{unit} to the one muon pair production corresponds to the exchange of light-by-light blocks between the two nuclei (Fig. 3). We start with a more general process — the production of one $\mu^+\mu^-$ pair and n electron-positron pairs ($n \geq 0$) in a collision of two ultra-relativistic nuclei

$$Z_1 + Z_2 \rightarrow Z_1 + Z_2 + \mu^+\mu^- + n(e^+e^-) \quad (37)$$

taking into account the unitarity corrections, which correspond to the exchange of the blocks of light-by-light scattering via the virtual lepton loops. The corresponding cross section $d\sigma_{1+n}$ can be calculated by a simple generalization of the results obtained in [6] for the n -pair process without muon pair production: $Z_1 + Z_2 \rightarrow Z_1 + Z_2 + n(e^+e^-)$. This multiple pair production process was studied in a number of papers, see [1] for a review. It was found that the probability is to a good approximation given by a Poisson distribution with the deviations found to be small. Indeed, it is not difficult to show that the basic equations for the latter process should be modified as follows. In Eq. (26) of [6] the additional factor

$$\tilde{B}^\mu(\boldsymbol{\rho}, \mathbf{r}_{n+1}) e^{-L[A^\mu(\rho)/2 + i\varphi^\mu(\rho)]} \quad (38)$$

appears under the integral, where $L = \ln(\gamma_1\gamma_2)$ and the functions \tilde{B}^μ , A^μ and φ^μ are the same as the functions \tilde{B} , A and φ in Eq. (27) of [6] but with the replacement of electrons by muons. As a result, Eq. (31) of [6] is replaced by

$$\frac{d\sigma_{1+n}}{d^2\rho} = LA_1^\mu(\rho) \frac{[LA_1(\rho)]^n}{n!} e^{-LA_1^\mu(\rho) - LA_1(\rho)}, \quad (39)$$

where $LA_1^\mu(\rho) \approx P_B(\rho)$ is the probability for one muon pair production in the Born approximation, as discussed in Sect. 2.2. In the region of interest, $\rho > 2R$, the function $A_1^\mu(\rho)$ is small,

$$LA_1^\mu(\rho) \ll 1, \quad A_1^\mu(\rho) \ll A_1(\rho), \quad (40)$$

therefore, we can rewrite (39) in the simpler form

$$\frac{d\sigma_{1+n}}{d^2\rho} = P_{1+n}(\rho), \quad P_{1+n}(\rho) = P_B(\rho) \frac{[\bar{n}_e(\rho)]^n}{n!} e^{-\bar{n}_e(\rho)}, \quad (41)$$

where $\bar{n}_e(\rho) = LA_1(\rho)$ is the average number of e^+e^- pairs produced in collisions of the two nuclei at a given impact parameter ρ . The result that the probabilities for the different processes factorizes is due to the independence of the individual processes. For a general discussion of the validity of this factorization together with possible violations we refer to [10].

In particular, we get the cross section for the exclusive one $\mu^+\mu^-$ pair production including the unitarity correction as

$$\sigma_{1+0} = \int P_B(\rho) e^{-\bar{n}_e(\rho)} d^2\rho. \quad (42)$$

This expression can be rewritten in the form

$$\sigma_{1+0} = \sigma_B + \sigma_{\text{unit}}, \quad (43)$$

where

$$\sigma_B = \int P_B(\rho) d^2\rho \quad (44)$$

is the Born cross section discussed in Sect. 2 and

$$\sigma_{\text{unit}} = - \int [1 - e^{-\bar{n}_e(\rho)}] P_B(\rho) d^2\rho \quad (45)$$

corresponds to the unitarity correction for the one muon pair production.

A rough estimation of σ_{unit} can be done as follows. The main contribution to σ_{unit} comes from the region

$$R \ll \rho \ll 1/m_e \quad (46)$$

in which the function $\bar{n}_e(\rho) \approx \bar{n}_e(2R)$ and the integral (45) can be calculated in LLA. It gives

$$\sigma_{\text{unit}} \sim -\frac{28}{27\pi} \frac{(Z_1\alpha Z_2\alpha)^2}{\mu^2} [1 - e^{-\bar{n}_e(2R)}] J_{\text{unit}}, \quad (47)$$

where

$$J_{\text{unit}} = 6 \int_{2R}^{1/m_e} \Phi(\rho) \frac{d\rho}{\rho}. \quad (48)$$

As a result, we find $\sigma_{\text{unit}} \sim -1.2$ barn for the Pb-Pb collisions at LHC, which corresponds approximately to $(-50)\%$ of the Born cross section.

It is seen that unitarity corrections are large, in other words, the exclusive production of one muon pair differs considerable from its Born value.

IV. INCLUSIVE PRODUCTION OF ONE $\mu^+\mu^-$ PAIR

The experimental study of the exclusive muon pair production seems to be a very difficult task. Indeed, this process requires that the muon pair should be registered without any electron-positron pair production including e^\pm emitted at very small angles. Otherwise, the corresponding cross section will be close to the Born cross section.

To prove this, let us consider the process (37), whose probability is given by Eq. (41). The corresponding cross section is

$$\sigma_{1+n} = \int P_{1+n}(\rho) d^2\rho. \quad (49)$$

It is clearly seen from this equation that after summing up over all possible electron pairs we obtain the Born cross section

$$\sum_{n=0}^{\infty} \sigma_{1+n} = \sigma_B. \quad (50)$$

Therefore, there is a very definite prediction: the inclusive muon pair production coincides with the Born limit. This direct consequence of calculations, which take into account strong field effects, may be easier to test experimentally than the prediction for cross sections of several e^+e^- pair production.

V. TWO MUON PAIR PRODUCTION

The cross section of the process

$$Z_1 Z_2 \rightarrow Z_1 Z_2 + \mu^+ \mu^- \mu^+ \mu^- \quad (51)$$

can be calculated in lowest order in α according to

$$\sigma_2 = \frac{1}{2} \int [P_B(\rho)]^2 d^2\rho \quad (52)$$

with the integration region $\rho \geq 2R$. But in this region the probability $P_B(\rho)$ is given to a good accuracy by Eqs. (23)–(25). From this we get $\sigma_2 = 1.24$ mbarn for Pb-Pb collisions at LHC.

VI. CONCLUSION

The exclusive production of one $\mu^+\mu^-$ pair in collisions of two ultra-relativistic nuclei is considered. We present a simple method for the calculation of the Born cross section

for this process.

We found that the Coulomb corrections to this cross section are small, while the unitarity corrections are large. This is in sharp contrast to the exclusive e^+e^- pair production where the Coulomb corrections to the Born cross section are large while the unitarity corrections are small.

We calculate also the cross section for the production of one $\mu^+\mu^-$ pair and several e^+e^- pairs in LLA. Using this cross section we found that the inclusive production of $\mu^+\mu^-$ pair coincides in this approximation with its Born value.

Let us discuss the relation of the cross sections obtained for the muon pair production with the differential cross section of the e^+e^- pair production in the region of large transverse momenta for the e^\pm , for example at $p_{\pm\perp} \gtrsim 100$ MeV. It is clear that for the e^+e^- pair production in this region, the situation is similar to the case considered for $\mu^+\mu^-$ pair production.

We expect that the inclusive production of a single e^+e^- pair with large transverse momenta of e^\pm (together with several unobserved e^+e^- pairs in the region of small transverse momenta of e^\pm of the order of m_e) coincides with the Born limit.

Appendix

To tests the approach used in Sect. 2.3, we consider the simpler case of the muon pair production by a real photon with the energy ω off an nucleus

$$\gamma Z \rightarrow Z \mu^+ \mu^- . \quad (53)$$

This cross section was calculated by Ivanov and Melnikov in Ref. [11] using the same expression (26) for the form factor of the nucleus and assuming $\Lambda^2/(2\mu)^2 \ll 1$. The corresponding formula for the Born contribution and the first Coulomb correction is

$$\sigma_{\gamma Z} = \frac{28}{9} \frac{Z^2 \alpha^3}{\mu^2} (l - C_1 - C_2) , \quad (54)$$

where

$$l = \ln \frac{2\omega\Lambda}{\mu^2} - \frac{57}{14} , \quad C_1 = \frac{12}{35} \left(\frac{\Lambda}{2\mu} \right)^2 , \quad C_2 = 0.92 (Z\alpha)^2 C_1 . \quad (55)$$

Therefore, the relative magnitude of the Coulomb correction is given by

$$\eta_2 = \frac{C_2}{l} , \quad (56)$$

which confirms the estimate in (36).

In the equivalent photon approximation, the cross section is given by

$$d\sigma_{\gamma Z}^{\text{EPA}} = dn_2 \sigma_{\gamma\gamma}(W^2) \quad (57)$$

which has the form

$$\sigma_{\gamma Z}^{\text{EPA}} = \frac{Z^2\alpha}{\pi} \int_{4\mu^2}^{\infty} \frac{dW^2}{W^2} f\left(\frac{W^2}{2\omega\Lambda}\right) \sigma_{\gamma\gamma}(W^2). \quad (58)$$

The main contribution to this integral is given by the region near the lower limit, where the argument of the function f is small and therefore f can be replaced by its approximate expression (29):

$$f\left(\frac{W^2}{2\omega\Lambda}\right) = 2 \ln \frac{\omega\Lambda}{2\mu^2} - 2 - 2 \ln \frac{W^2}{4\mu^2}. \quad (59)$$

After that the cross section can be calculated without difficulties as

$$\sigma_{\gamma Z}^{\text{EPA}} = 2 \frac{Z^2\alpha}{\pi} \left[\left(\ln \frac{\omega\Lambda}{2\mu^2} - 1 \right) I - I_1 \right] = \frac{28}{9} \frac{Z^2\alpha^3}{\mu^2} l, \quad (60)$$

where I is given by (15) and

$$I_1 = \int_{4\mu^2}^{\infty} \frac{dW^2}{W^2} \sigma_{\gamma\gamma}(W^2) \ln \frac{W^2}{4\mu^2} = \frac{(43 - 28 \ln 2) \pi \alpha^2}{9\mu^2}. \quad (61)$$

Comparing this expression with the one of (54), we find that those terms, which are omitted in the EPA, have a relative magnitude of the order of

$$\eta_1 = \frac{C_1}{l}, \quad (62)$$

this expression confirms the estimate (34).

Acknowledgments

We are grateful to D. Ivanov and A. Milstein for useful discussions. This work is supported by RFBR (code 06-02-16064).

-
- [1] G. Baur, K. Hencken, D. Trautmann, S. Sadovsky, Yu. Kharlov. Phys. Rep. **364**, 359 (2002).
 - [2] L.D. Landau, E.M. Lifshitz, Phys. Z. Sowjet. **6**, 244 (1934); G. Racah, Nuovo Cimento **14**, 93 (1937).

- [3] S.R. Klein. Nucl. Instrum. Meth. A **459**, 51 (2001) [arXiv:physics/0005032].
- [4] D.Yu. Ivanov, A. Schiller, V.G. Serbo, Phys. Lett. B **454**, 155 (1999).
- [5] R.N. Lee, A.I. Milstein, V.G. Serbo, Phys. Rev. A **65**, 022102-1 (2002); Proc. of the Int. Conference on the Structure and Interactions of the Photon, *PHOTON 2001* p.339, (Ascona, Switzerland, 2–7 September 2001). Eds. M.-N. Kienzle-Focacci and M. Wadhwa (World Scientific, New Jersey-London-Singapore-Hong Kong, 2002).
- [6] E. Bartoš, S.R. Gevorkyan, E.A. Kuraev, N.N. Nikolaev. Phys. Lett. B **538**, 45 (2002) and arXiv:hep-ph/0204327.
- [7] K. Hencken, E.A. Kuraev, V.G. Serbo, Acta Physica Polonica B **37**, 969 (2006) and arXiv:hep-ph/0510360.
- [8] V. M. Budnev, I. F. Ginzburg, G. V. Meledin, V. G. Serbo, Phys. Rep. **15C**, 181 (1975).
- [9] K. Hencken, D. Trautmann, G. Baur, Phys. Rev. A **51**, 1874 (1995).
- [10] G. Baur, K. Hencken, A. Aste, D. Trautmann and S.R. Klein. Nucl. Phys. A **729**, 787 (2003) [arXiv:nucl-th/0307031].
- [11] D. Ivanov, K. Melnikov. Phys. Rev. D **57**, 4025 (1998).